Decoupling Controller Design Based on Gain and Phase Margin Specifications for a Coupled Tank System Model

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based on research carried out

under the supervision of

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May, 2016

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May 25, 2016

Supervisor's Certificate

This is to certify that the work presented in the dissertation entitled *Decoupling Controller Design Based on Gain and Phase Margin Specifications for a Coupled Tank System Model* submitted by *Oindrilla Chakraborty*, Roll Number 214EE3231, is a record of original research carried out by her under my supervision and guidance in partial fulfillment of the requirements of the degree of *Master of Technology* in *Electrical Engineering*. Neither this dissertation nor any part of it has been submitted earlier for any degree or diploma to any institute or university in India or abroad.

Sandip Ghosh

Dedication

To my beloved Parents (Ma, Baba) and sister (Oisikha)

Oindrilla Chakraborty

Declaration of Originality

I, *Oindrilla Chakraborty*, Roll Number *214EE3231* hereby declare that this dissertation entitled *Decoupling Controller Design Based on Gain and Phase Margin Specifications for a Coupled Tank System Model* presents my original work carried out as a postgraduate student of NIT Rourkela and, to the best of my knowledge, contains no material previously published or written by another person, nor any material presented by me for the award of any degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the dissertation. Works of other authors cited in this dissertation have been duly acknowledged under the sections “Reference” or “Bibliography”. I have also submitted my original research records to the scrutiny committee for evaluation of my dissertation.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present dissertation.

May 25, 2016
NIT Rourkela

Oindrilla Chakraborty

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Abstract

The objective of a multi-variable control involves maintaining various control variables at independent set points. The interactions present in the system affects more than one controlled variables because of the manipulated variable. Decouplers are designed to reduce the interactions in between the loops in to achieve a satisfactory responses when there is presence of non-minimum phase zeros,multiple time delays and large uncertainty. The dynamic and static decoupling are the two types of decoupling strategies. In this thesis, these control strategies are discussed. In practice, there exists certain process unmodelled dynamics. Hence , there is a necessity to examine the robust stability of a system to check whether the control system stability is ascertained in presence of these unmodelled dynamics.

This thesis deals with designing a controller along with decoupler to achieve the desired performance of a TITO system. At first, a decoupler is being designed from the plant matrix. Then, a first order plus dead time model is obtained for each of the decoupled process on the basis of the frequency response fitting. After getting the FOPDT model a decentralized PI/PID controller for each reduced order decoupled model is designed to obtain desired gain and phase margins.

The present technique is applied to a coupled tank system.The characteristics like non-minimum phase and non-linear characteristics make the control of coupled tank liquid level system, a standout amongst the most difficult benchmark control problems. The main objective of the coupled tank system is to maintain a desired level of liquid in the two tanks independent of each other when the water enters the tank and when the water flows out. The coupling impact here in this framework is a coupling switch that permits stream of water in the tank at higher level to a tank at lower level.

Lastly, robust stability of the control system is analyzed in the presence of various process uncertainties like additive uncertainty and multiplicative uncertainties. The stability analysis is examined using the small gain theorem or the spectral radius criterion. The robust stability of the coupled tank system is also determined.

***Keywords*: *Multi-variable systems*; *Decentralized control*; *Gain margin*; *Phase margin*; *Uncertain systems*; *Robust stability* .**

Contents

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List of Acronyms

TITO	:	Two-Input Two-Output
MIMO	:	Multi-Input Multi-Output
SISO	:	Single-Input Single
RHP	:	Right Half Plane
GM	:	Gain Margin
PM	:	Phase Margin
IMC	:	Internal Model Control
TFM	:	Transfer Function Matrix
FOPDT	:	First Order Plus Dead - Time

Chapter 1

Introduction

The main objective of a control system is to actuate a given set of process variables to behave in a desired manner either by satisfying the needs of time or frequency domain or by accomplishing the best performances as expressed by an optimization index [1]. The process designed by a process engineer is in accordance to the best of their knowledge in that field and some assumptions in the operating system. This process will later on operate under certain other conditions that support external disturbances usually not well better-known or determined [1].

The characteristics of the process will vary with time or with the variation in the load. The main goal of the control system is to cope up with these changes and provide a suitable behavior. Industrial processes are described as multi-variable system by the control engineers. In process industry most of the operating units are required to have control over product rate and its quality by adjusting the inputs to the process, thereby they are referred to as multi-variable system.

1.1 Multi-variable Systems

In a typical system, there are several system variables that are need to be controlled and hence they are called as multi-variable system. In various chemical industries, multi-variable systems can be found in a chemical reactor, distillation columns and heat exchanger. An example of a multi-variable system can be illustrated in an air cooling system where temperature and humidity are to be controlled; missile tracking in a military operation; in an aeroplane where the angle of deviation and the speed needs to be controlled.

Multi-variable system can be represented in state space form with m outputs and n inputs (for a linear system) as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1.1}$$

$$y(t) = Cx(t) + Du(t) \tag{1.2}$$

where $A \in \Re^{[p,p]}$,$B \in \Re^{[p,n]}$, $C \in \Re^{[m,p]}$, $D \in \Re^{[m,n]}$, $x(t)$ is the state vector, $u(t)$ is the input (or control) vector and $y(t)$ is the output vector . We can represent a p variable system as shown below:

$$Y(s) = G(s)U(s) \tag{1.3}$$

$$\begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_p(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1p}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2p}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{p1}(s) & g_{p2}(s) & \cdots & g_{pp}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_p(s) \end{bmatrix} \tag{1.4}$$

1.2 Different Control Strategies of Multi-variable System

The objective of MIMO system is to maintain several controlled variables at individual set point. The cross coupling in the system between the outputs and the inputs will cause the manipulated variable to affect more than one controlled variable. The different control schemes are as follows.

1. Decentralized Structure
2. Centralized Structure
3. Decoupled Structure

1.2.1 Decentralized Structure

The characteristics of decentralized structure as shown in figure 1.1 is as follows [1]:

1. The main aim is to disintegrate a system into subsystems and design a individual controller for each subsystem.
2. Decentralized controllers have a few advantages that they can be implemented and tuned easily if there is any variation in the process conditions.
3. One advantage of a decentralized controller over the centralized controller is that the tolerance,due to failure in the manipulated variable, can be easily incorporated. this means that the system will maintain its stability even though any one of the controller is not working and the tuning will be easier as there will be less number of parameters.
4. If a controller goes out of service, then it does not affect other loops and thereby preserving the stability.

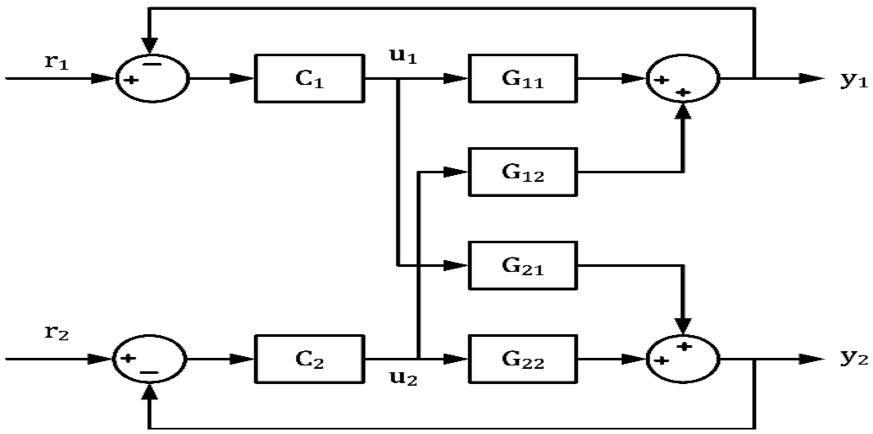

Figure 1.1: Block Diagram of Decentralized Structure

1.2.2 Centralized Structure

The characteristics of centralized structure as shown in figure 1.2 is as follows [1]:

1. One of the advantage of this control strategy is that only with the information of the steady state gain matrix, the multi-variable PI controller is easy to tune and hence easily designed [1].

2. $'n'$ manipulated variables are used to control the $'n'$ no. of output variables.These controllers are not diagonal in form.

3. One of the disadvantage of this strategy is scheme is that the controller matrix has complex calculations and to understand the control loops is difficult.

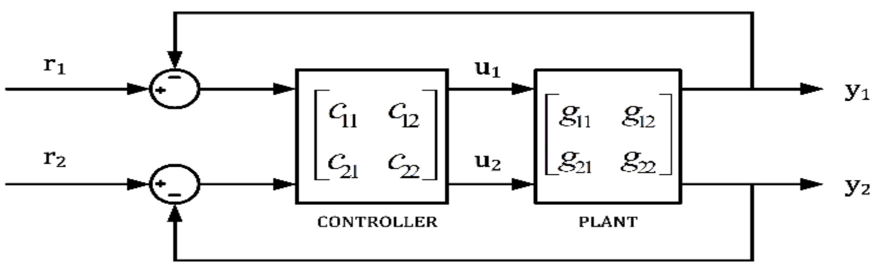

Figure 1.2: Block Diagram of Centralized Structure

1.3 Decoupling Structure

The characteristics of decoupling structure as shown in figure 1.3 are as follows:

1. If the multi-loop control cannot reach the desired specification, then decoupling is done to tackle the MIMO control and the transfer function matrix is transformed into a diagonal one [1, 2].

2. Decoupling is done to reduce the interactions between the loops. As the MIMO system is disintegrated into several subsystems , there should be a proper selection of the input and output variables [3].

3. The disadvantage of centralized strategy , that is complexity in the calculation of the structure,and decentralized strategy, that is pairing of input-output variables, are all taken into account in this strategy.

4. Online adjustment of the control variables is difficult. The variables have to be adjusted in real time and hence there arises a necessity to decouple the MIMO system into several subsystems with very strong interactions.

5. Decoupling could be achieved in two different ways:
   - Feed-forward
   - Feedback

6. Niederlinski index, RGA analysis, singular value decomposition, Dynamic RGA etc is required to specify the nature of interactions present in the system.

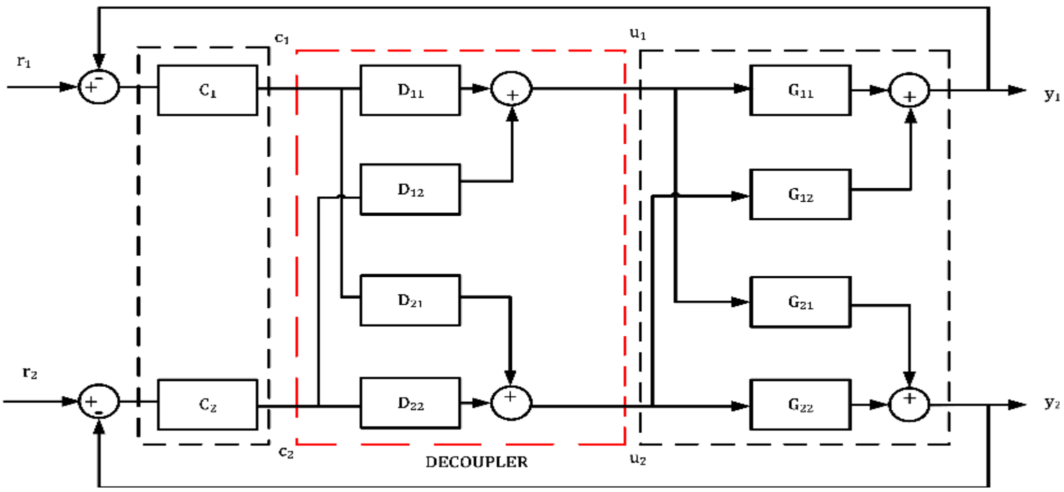

Figure 1.3: Block Diagram of Decoupling Structure

1.4 Robust stability analysis

There are different significant issues that surpass the limits of a specific applications in designing a control system. In spite of the fact that they might contrast along with every application and might also have distinctive stages of significance, these outcomes are non specific in their connection to control design targets and strategies. Key to these outcomes is the prerequisite to give acceptable execution even with modeling errors, instability and system variations [4].

Generally, most of the control designs depend on the utilization of a certain desired model. The relation between the actual plant and the model is very complex. The nature of a model relies on upon how accurately its output responses coordinate with those of the actual plant. Since no single fixed model can react precisely like the actual plant, hence proper arrangements are required to solve these problems. However we can actually find that the mathematical model of a plant is very different from that of the actual physical model. Hence one can never obtain a model with the same characteristics as that of the true plan.

Hence the control engineer has to make the proper design of the mathematical model. Hence the acceptable model of the plant should must be simple enough to alleviate the design, and it must be so complex that the engineer himself is assured that the particular model designed is proper enough to operate for the new plant [4].

The word uncertainty describes the difference between the actual plant and its model. To determine the robust stability, under many assumptions of the model uncertainties, the small gain theorem is applied. The error Δ is considered as stable. The uncertainty represented by Δ is designed such that if it crosses this limitation then the system will lead to instability. From the small gain theorem we will find that $\|\Delta\|_\infty$ must be larger than $\|M\|_\infty$ [5] .

1.5 Literature Review

Earlier industries used to operate manually. There were a lot of disadvantages in manual operation like constant check on the variables was must , otherwise there will be many observational errors. In any industry, there would be many parameters that would be varying with time. The cost of the equipments, the requirement of high accuracy and precision, requirement of skilled labour was not economical and feasible because of the growing industrialization.With the growth of industries, there was advancement in the automatic control applications. A processes were controlled automatically. The controller became the key constituent of the process industry. The advantage of designing the controller was that it could curb the disadvantages following manual operations. The controller could be hardware or any software code. The controller has to receive the information from the sensors, then process it, and give the appropriate commands to obtain the desired response [5].

In an industry, there will be several process variables that are required to be controlled and can be defined as multi-variable systems. These systems will have a lot of interactions present because of the variations in the parameters. The interactions are such that any change in a specific parameter will affect the responses of other process parameters. In order to minimize these interactions or cross coupling, the system is disintegrated into several subsystems. Then the process will be called as a decoupled process. Proper decoupling will lead to a better performance of the system.

The industrial process poses large time delays in general. They forbid the high gain of the closed loop controller [14], sluggish response and offset. These time delays poses a lot of difficulties for the multi-variable systems as there are multiple time delays. Several strategies have been formulated to control the TITO systems with time delays. In [6], closed loop design strategy have been given in the conventional unity feedback control structure. The smith predictor approach applied to the SISO system [7] having time delay, can be applied to the multi-variable system. In chemical industries, the multi-variable systems are treated as TITO system. This SP method can be applied here to the TITO processes such that there is a time-delay free characteristic equation. This method had a drawback as the designing of the controller was not possible. From the researches done, it was found that the dynamic decoupling was difficult to configure for the systems with large time delays, as discussed in [8].

In [9], here in this article an new method had been proposed under IMC decoupling technique for the TITO system having time delays. This is any analytical decoupling approach in the IMC structure. The tuning of the controller was directed to incorporate the cross-coupling between the individual loop and the system performance discussed in [10]. This tuning was done because of the presence of non-minimum phase zeros and time-delays. This method limits the possible interactions in the multi-variable system. Hence a remarkable advancement was observed in the decoupling regulation. Here a controller is designed depending upon the required diagonal transfer matrix. This diagonal transfer matrix is in terms of the robust optimal performance objective [14]. Also, the robust stability analysis was analyzed under the presence of uncertainty that we come across in practice. Similarly in [11] also the desired diagonal transfer matrix was suggested to determine the controller transfer matrix. To make the difficulties that is dealt with during the implementation less severe, various cases are taken to design the controller matrix like, infinite RHP zeros but finite LHP zeros, RHP and LHP zeros are infinite, finite RHP zeros and no RHP zeros. Here also the robust stability analysis was performed in case of uncertainties like additive, multiplicative input and multiplicative output.

In [12], a relative study on simplified , ideal and inverted decoupling was proposed. The simplified decoupling is well-known method. The ideal decoupling is very rarely used method. the disadvantage of ideal decoupling is its perceptance to modelling errors. The inverted decoupling generally serves the advantages of the above decoupling methods [13]. The robust stability and performance were analyzed for all the three decoupling techniques as the controller is tuned so that desired closed loop nominal performances are obtained.

In [14], for a TITO process a decentralized PI/PID controller is designed depending upon the gain margin (GM) and phase margin (PM). Generally we represent a multi-variable system as TITO process as there exists such processes that are of this type and many of the higher order processes are decomposed into TITO systems with minimal interactions within the output and input [15–17]. Here simple decoupler was designed having a decentralized PI/PID controller. The diagonal decoupler thus obtained was reduced into an FOPDT model by the frequency response model order reduction method. The PI/PID controller was designed based on the gain margin and phase margin specification.

1.6 Thesis Organization

- **Chapter 2:** Here different decoupling techniques are described.
- **Chapter 3:** Robust stability analysis of any system is discussed, also different uncertainties are described briefly.
- **Chapter 4:** This chapter describes the Model Reduction Technique Using Gain and Phase Margin.
- **Chapter 5:** This chapter describes the coupled tank system and the model reduction techniques is applied.
- **Chapter 6:** Conclusion and future scope.

Chapter 2

Different Decoupling Techniques

This chapter describes the need of decoupling and different decoupling techniques that are applied to a multi-variable system. these decoupling methods were analyzed by an illustrative example.

2.1 Introduction

There are many undesirable cross-coupling in industrial processes. This is a control problem today. All these processes are multi-variable in nature. It is found that managing the MIMO system is quite difficult than the SISO system but properties of both SISO and MIMO systems are almost analogous [18, 19].

The outcome is that multi-variable control strategies created amid the most recent a quarter century affected genuine applications. As we know that there are a lot of loop interactions in the multi-variable systems, these interaction or cross couplings needs to be reduced otherwise these cross coupling may affect the performance of the plant. Therefore in order to reduce these interactions, decouplers are applied in the plant. The decouplers break up a system into various subsystems.Once the plant is decoupled, the set-point change in one of the process variables affects the response to that process variables and no other process variable is affected [5].

2.2 The Decoupling Problem

The multi-variable processes can be described by time delays, large uncertainties and non.minimum phase [3]. There are several design techniques for a SISO system, but these methodologies are not applicable for the MIMO systems since they posses a lot of interactions. A multi-variable system has a drawback, that is, if we try to change any parameter of the controller it will affect that particular loop and also other loops. This may lead to damage in the whole system.

2.3 Different Types of Decoupling Techniques

There are different types of decoupling technique applicable to multi-variable system. They are classified based on the time characteristics as static decoupling and dynamic decoupling [6]. Consider a MIMO system as shown in 1.3 i.e

$$Y(s) = G(s)U(s)$$

1. **Static Decoupling :** If G(0) is stable and diagonal, non singular, then this type of decoupling method is called as static decoupling.

2. **Dynamic Decoupling :** If G(s) is stable and diagonal at all frequencies, then this type of decoupling is called as dynamic decoupling.

Dynamic decoupling is a bit complex in nature, the control law is highly sensitive. We know that there exists a lot of interactions in the multi-variable systems. There may be systems where it is observed that the coupling is firm between the plant variables all over the operating frequency. Dynamic decoupler is applicable for such conditions. There are different techniques for dealing with the dynamic deoupling strategy : internal model approach, simplified decoupling, ideal decoupling, inverted decoupling.

Multi-variable systems are expressed as a TITO system in practice. From the figure shown below, it can be observe that we can design a multi-variable controller having the characteristics of decoupling a system and also controlling it. Apart from this we can also separately design a decoupler that would disintegrate the system into several subsystems and applying any SISO methodology for designing a controller.

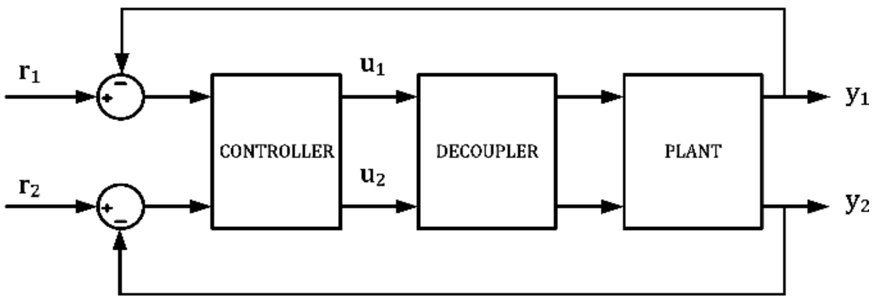

Figure 2.1: Dynamic Decoupling Representation

2.3.1 Internal Model Control

There are a few techniques that are exhibited for tuning PID controllers and built up a model-based strategy to combine a controller that yield required closed loop response. We can say that control system is stable if the controller and the process is stable [19]. Here in the IMC based controller design it follows the internal model principle. This principle states

that accurate control can be observed if the controller explicitly or implicitly represents the process that needs to be controlled [20] . In this method [9], the open loop transfer function is considered.
The figure below shows the basic representation of an IMC structure.

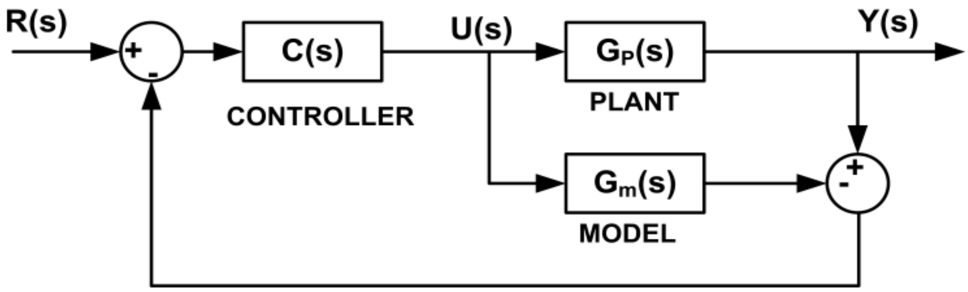

Figure 2.2: IMC Structure Representation

The transfer matrix of the system can be represented as

$$H(s) = G_p(s)C(s)[I + (G_p(s) - G_m(s)C(s)]^{-1} \tag{2.1}$$

The 2.1 can be complex and has a chance to lose its stability. Hence, we need to design a controller analytically such that it represents the reference model to obtain the desired response. In this method, we can make two assumptions in designing a controller; one is when $G_p(s) = G_m(s)$ and second is when $G_p(s) \neq G_m(s)$. The TFM H(s) is considered as a diagonal one.

$$H(s) = \begin{bmatrix} h_1(s) & 0 & \cdots & 0 \\ 0 & h_2(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_n(s) \end{bmatrix} \tag{2.2}$$

where $h_i(s)$ must be stable and proper; i=1,2... n . If $det(G_p) \neq 0$, then only $det(H) \neq 0$ is satisfied. Suppose $G_p(s) = G_m(s)$, then the TFM equation will be $H(s) = G_p(s)C(s)$. The controller can be obtained as shown below:

$$C = \frac{adj(G_p)}{det(G_p)} H \tag{2.3}$$

The values of H(s) is chosen as per the presence od RHP zero.The standard form of h_i is -

$$h_i(s) = \frac{e^{-\theta_i s}}{\lambda_i s + 1} \prod_{i=1}^{n} \left(\frac{-s + z_i}{s + z_i^*} \right) \tag{2.4}$$

where z_i is the RHP zero and z_i^* is the complex conjugate. The controller is designed is based are on the above equations and assumptions. There may be a case when the plant and the model have certain differences in their parameters so large enough to make the plant unstable and desired results cannot be obtained.

Therefore, there is one more method to design a controller analytically [11]. Here in this method closed loop transfer function of the multi-variable system is considered to design the controller. The control structure will be as shown below:

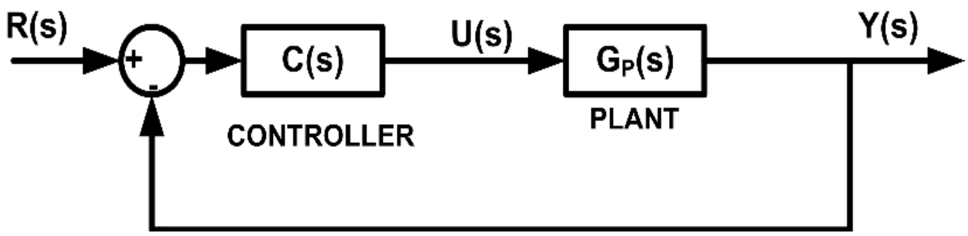

Figure 2.3: Generalized Control Structure

The transfer function matrix (TFM) would be [9]:

$$H = G_p C[I + G_p C]^{-1} \tag{2.5}$$

From the 2.5, controller matrix determined would be

$$C = G_p^{-1}[H^{-1} - I] = \frac{adj(G_p)}{det(G_p)} \frac{h_{ii}}{1 - h_{ii}} \tag{2.6}$$

From these methods discussed above, we observe that:

1. Only a adjustable tuning parameter (λ_i) can tune the controller matrix.
2. While tuning any of the controllers there is no interactions between the variables.
3. This approach is based on the centralized control strategy.
4. The limitation of this approach is that the controller can become complex for higher order system.

2.3.2 Simplified Decoupling and Inverted Decoupling

Considering a multi-variable system represented as a TITO system, with $G_p(s)$ process transfer matrix, $D(s)$ is the decoupling transfer matrix. Let T(s) be the diagonal transfer matrix [12], such that

$$D(s) = \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix}, \; G_p(s) = \begin{bmatrix} G_{p11}(s) & G_{p12}(s) \\ G_{p21}(s) & G_{p22}(s) \end{bmatrix}, \tag{2.7}$$

$$T(s) = G_p(s)D(s) = \begin{bmatrix} T_{11}(s) & 0 \\ 0 & T_{22}(s) \end{bmatrix} \tag{2.8}$$

The controller matrix would be:

$$C(s) = \begin{bmatrix} c_1(s) & 0 \\ 0 & c_2(s) \end{bmatrix} \tag{2.9}$$

Substituting 2.7 into 2.8 , we get:

$$D(s) = G_p(s)^{-1}T(s) = \frac{1}{G_{p11}(s)G_{p22}(s) - G_{p12}(s)G_{p21}(s)} \begin{bmatrix} G_{p22}(s)T_{11}(s) & -G_{p12}(s)T_{22}(s) \\ -G_{p21}(s)T_{11}(s) & G_{p11}(s)T_{22}(s) \end{bmatrix} \tag{2.10}$$

Simplified decoupling

The simplified decoupling is described by selecting the decoupler as [12]:

$$D(s) = \begin{bmatrix} 1 & -\frac{G_{p12}(s)}{G_{p11}(s)} \\ -\frac{G_{p21}(s)}{G_{p22}(s)} & 1 \end{bmatrix} \tag{2.11}$$

The transfer matrix that will be obtained is:

$$T(s) = \begin{bmatrix} G_{p11}(s) - \dfrac{G_{p12}(s)G_{p21}(s)}{G_{p22}(s)} & 0 \\ 0 & G_{p22}(s) - \dfrac{G_{p12}(s)G_{p21}(s)}{G_{p11}(s)} \end{bmatrix} \tag{2.12}$$

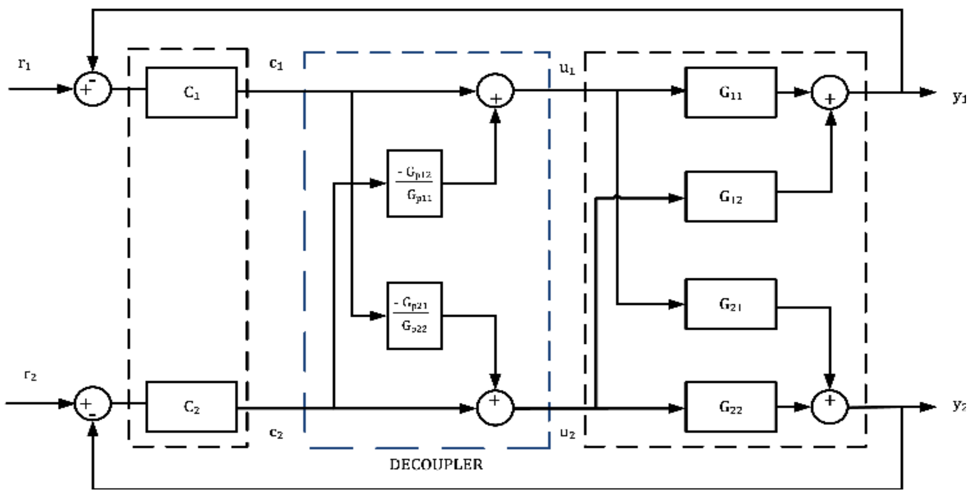

Figure 2.4: Simplified Decoupling

Inverted Decoupling

From the figure 2.5 and and 2.10, we get [12] :

$$u_1(s) = \left[\frac{G_{p11}(s)G_{p22}(s)}{G_{p11}(s)G_{p22}(s) - G_{p12}(s)G_{p21}(s)}\right] c_1(s) - \left[\frac{G_{p12}(s)G_{p22}(s)}{G_{p11}(s)G_{p22}(s) - G_{p12}(s)G_{p21}(s)}\right] c_2(s) \tag{2.13}$$

$$u_2(s) = -\left[\frac{G_{p21}(s)G_{p11}(s)}{G_{p11}(s)G_{p22}(s) - G_{p12}(s)G_{p21}(s)}\right] c_1(s) + \left[\frac{G_{p11}(s)G_{p22}(s)}{G_{p11}(s)G_{p22}(s) - G_{p12}(s)G_{p21}(s)}\right] c_2(s) \tag{2.14}$$

The above equations can be rewritten as:

$$u_1(s) = c_1(s) - u_2(s)\frac{G_{p12}(s)}{G_{p11}(s)} \tag{2.15}$$

$$u_2(s) = c_2(s) - u_1(s)\frac{G_{p21}(s)}{G_{p22}(s)} \tag{2.16}$$

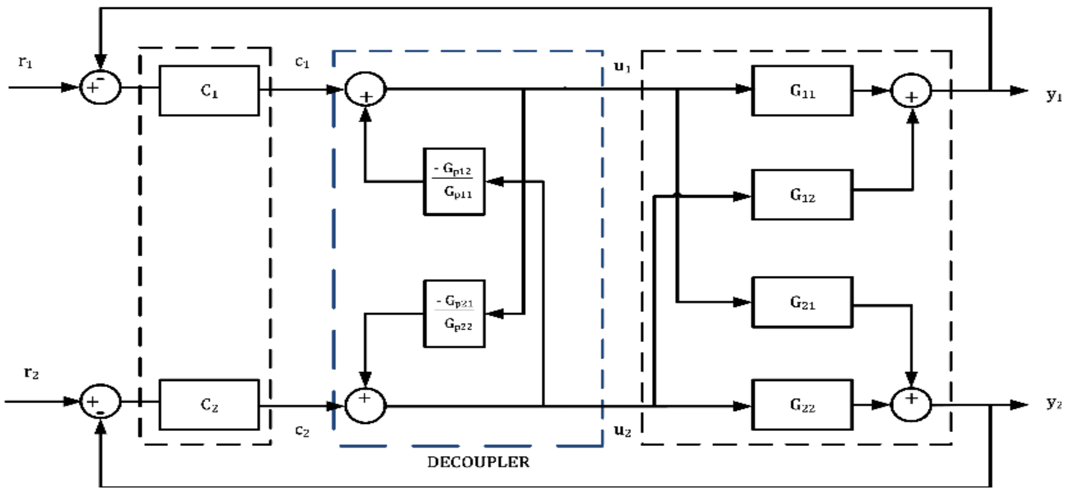

Figure 2.5: Inverted Decoupling

2.4 Illustrative Example

Considering a TITO (Two-Input-Two-Output) process with time delays of a Wood-Berry Distillation Process [11]

$$G = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{2.17}$$

2.4.1 IMC Approach

The centralized controller designed from this approach:

$$C = F \begin{bmatrix} \dfrac{16.7s+1}{12.8(\lambda_1 s+1)} & \dfrac{-0.0761(16.7s+1)(14.4s+1)e^{-2s}}{(21s+1)(\lambda_2 s+1)} \\ \dfrac{0.0266(16.7s+1)(14.4s+1)e^{-4s}}{(10.9s+1)(\lambda_1 s+1)} & \dfrac{-(14.4s+1)}{19.4(\lambda_2 s+1)} \end{bmatrix} \tag{2.18}$$

where

$$F = \frac{1}{1 - \dfrac{0.5023(16.7s+1)(14.4s+1)}{(21s+1)(10.9s+1)}e^{-6s}} \tag{2.19}$$

and $\lambda_1 = 4$ and $\lambda_2 = 6$

2.4.2 Inverted Decoupling

The decoupler determined is as shown below:

$$\text{D(s)} = \begin{bmatrix} 1 & \dfrac{-1.4766(16.7s+1)e^{-2s}}{(21s+1)} \\ \dfrac{-0.3042(14.4s+1)e^{-4s}}{(10.9s+1)} & 1 \end{bmatrix}$$

$$D(s) = \frac{1}{1 - \dfrac{0.5023(16.7s+1)(14.4s+1)}{(21s+1)(10.9s+1)}e^{-6s}} \begin{bmatrix} 1 & \dfrac{1.4766(16.7s+1)e^{-2s}}{(21s+1)} \\ \dfrac{0.3042(14.4s+1)e^{-4s}}{(10.9s+1)} & 1 \end{bmatrix} \tag{2.20}$$

The PI controller as determined from [21] will be:

$$C(s) = \begin{bmatrix} 0.7438 + \dfrac{0.0445}{s} & 0 \\ 0 & -0.141 - \dfrac{0.0098}{s} \end{bmatrix} \tag{2.21}$$

The comparative simulation results of the different decoupling techniques is as shown below :

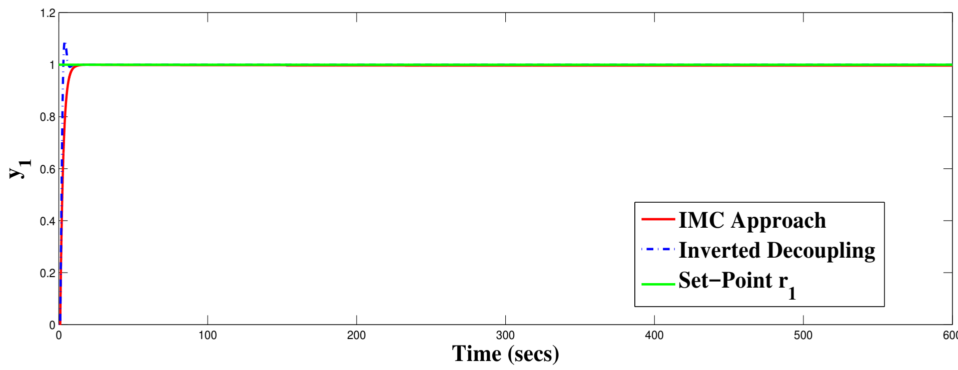

Figure 2.6: Output Response for y_1

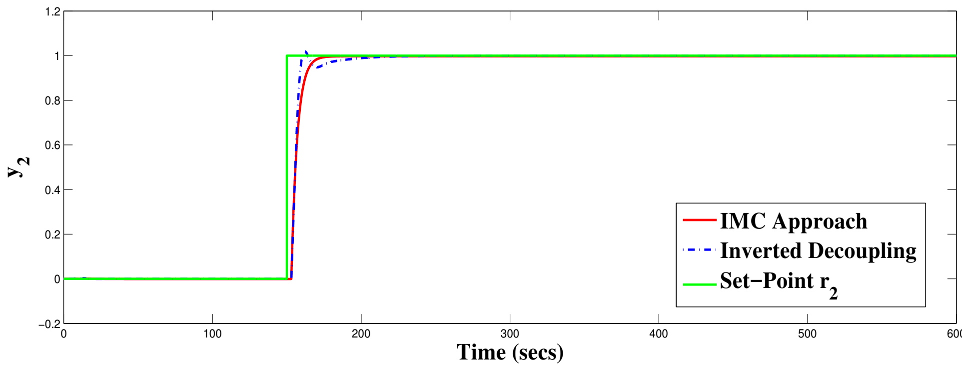

Figure 2.7: Output Response for y_2

From the results obtained, it was observed that :

- Initially at $t = 0$ seconds a unit step input is applied for the reference input 1 and $t = 150$ seconds for reference input 2.
- Both of the decoupling methods are capable of tracking the set-points effectively. There is only a slight difference in the rise time of both the decoupling methods.
- In case of the IMC approach, the choice of the tuning parameters should be proper enough for obtaining the desired output response. this is done through trial and error method.
- It was observed that in case of a inverted decoupling a small overshoot is present. This overshoot can vary if there is a slight change in the integral gain of the controller.

Chapter 3

Robust Stability Analysis

This chapter deals with various uncertainties in a physical system and representation of these uncertainties by any kind of perturbation.The main objective is to analyze the robust stability and performance of a MIMO systems with various perturbations.

3.1 Introduction

In the absence of any kind of external excitation , if all the signals in the system decay to zero, then it can be said that the system is stable. the stability of a closed loop system is a main requirement of the plant , as its absence will cause the signal to grow without any limitation, thereby destroying and breaking down the plant [22]. Practically, in engineering systems, it is very important to design control systems such that the stability is preserved in case of any kind of uncertainties. This property is commonly known as robust stability. A system is said to be robust if it is insensitive to any of the difference between the actual system and its reference model that is used for designing the controller [6].Hence, the robust stability of the control system is evaluated in the presence of any process uncertainty and thereby determining the tuning parameter that holds the stability.The robust stability of any system can be explained through the small gain theorem or the spectral radius stability criteria.

3.2 Methods For Robust Stability Analysis

3.2.1 Small Gain Analysis

The important tool that is used to analyze the stability of any closed loop perturbed system is the small gain theorem. This theorem states the condition under which the system with interconnected components is stable.

Consider a system as shown in figure 3.1

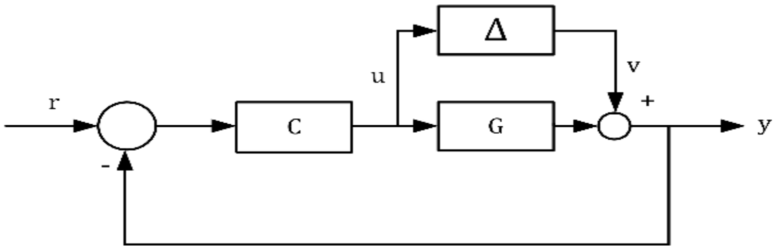

Figure 3.1: Process model with a uncertainty

This system is further rearranged to M-Δ structure for the robust stability analysis.

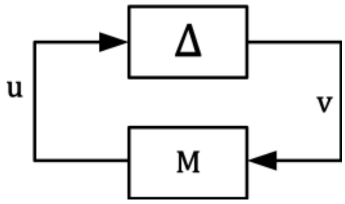

Figure 3.2: M-Δ Structure

According to the small gain theorem as described in [4, 6] any interconnected system as shown in fig with an uncertainty holds robust stability if and only if,

$$\|M(s)\Delta(s)\|_{\infty} < 1 \tag{3.1}$$

where M(s) is the closed loop transfer function and Δ represents the uncertainty of the system. Hence, we need to study the stability of M(s) and find the maximum limit of Δ for which the system is stable.

3.3 Plant Uncertainty

In general, there exist a process having unmodelled dynamics. Plant uncertainty arises from the inevitable discrepancies between the true plant and the model. In practice, there are three type of uncertainty encountered. These uncertainties are as follows:

3.3.1 Additive Uncertainty

The model uncertainty for a system as described in [6] can be represented as an additive perturbation as shown in figure 3.3. The process additive uncertainty can be referred to as parameter perturbations and their actual process family is $\prod_A = \{\hat{G}_A(s) : \hat{G}_A(s) = G(s) + \Delta_A\}$ where the Δ_A is assumed to be stable.

The M-Δ_A structure will be as shown in figure 3.4.

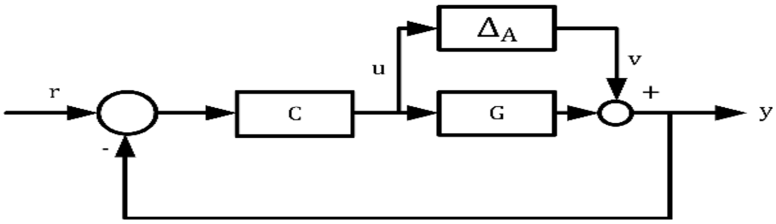

Figure 3.3: Process Additive Uncertainty

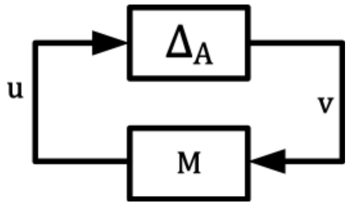

Figure 3.4: M-Δ Structure for Additive Uncertainty

We find $M_A(s)$ that is the transfer matrix from the output to input of Δ_A as shown in figure 3.5

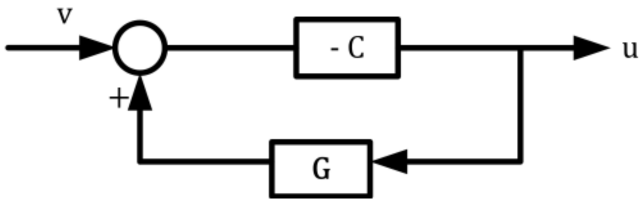

Figure 3.5: Block Diagram to find M(s)

$$M_A(s) = -C(s)[I + G(s)C(s)]^{-1} \tag{3.2}$$

3.3.2 Multiplicative Input Uncertainty

The model uncertainty for a system as described in [6] can be represented as an multiplicative input perturbation as shown in figure 3.6

The process additive uncertainty can be referred to as parameter perturbations and their actual process family is $\prod_I = \{\hat{G}_I(s) : \hat{G}_I(s) = G(s)(I + \Delta_I)\}$ where the Δ_I is assumed to be stable. The M-Δ_I structure will be as in figure 3.7

We find $M_I(s)$ that is the transfer matrix from the output to input of Δ_I as shown in figure 3.8

$$M_I(s) = -C(s)[I + G(s)C(s)]^{-1}G(s) \tag{3.3}$$

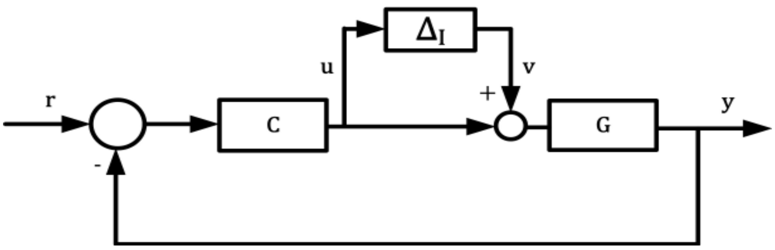

Figure 3.6: Process Multiplicative Input Uncertainty

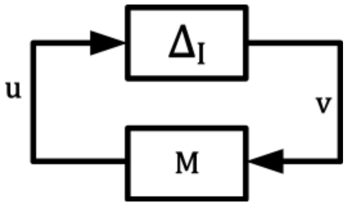

Figure 3.7: M-Δ Structure for Multiplicative Input Uncertainty

3.3.3 Multiplicative Output Uncertainty

The model uncertainty for a system as described in [6] can be represented as an multiplicative output perturbation as shown in figure 3.9.

The process additive uncertainty can be referred to as parameter perturbations and their actual process family is $\prod_O = \{\hat{G}_O(s) : \hat{G}_O(s) = (I + \Delta_O)G(s)\}$ where the Δ_O is assumed to be stable. The M-Δ_O structure will be as in figure 3.10

We find $M_O(s)$ that is the transfer matrix from the output to input of Δ_O as shown in figure 3.11

$$M_O(s) = -G(s)C(s)[I + G(s)C(s)]^{-1} \tag{3.4}$$

Therefore, based on the small gain theorem, the robust stability constraint can be obtained

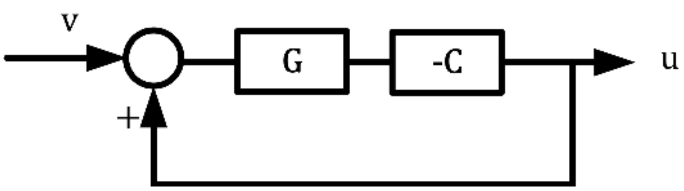

Figure 3.8: Block Diagram to find M(s)

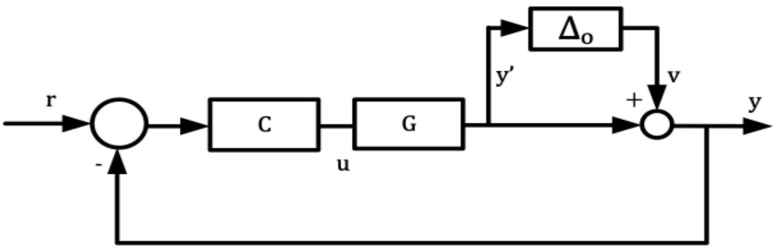

Figure 3.9: Process Multiplicative Output Uncertainty

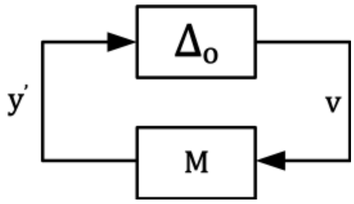

Figure 3.10: M-Δ Structure for Multiplicative Output Uncertainty

as follows:

$$\left\|C(I+GC)^{-1}\right\|_{\infty} < \frac{1}{\|\Delta_A\|_{\infty}} \tag{3.5}$$

$$\left\|C(I+GC)^{-1}G\right\|_{\infty} < \frac{1}{\|\Delta_I\|_{\infty}} \tag{3.6}$$

$$\left\|GC(I+GC)^{-1}\right\|_{\infty} < \frac{1}{\|\Delta_O\|_{\infty}} \tag{3.7}$$

From (3.5),(3.6) and (3.7) the robust stability constraints are not analytical and hence the computation effort for H infinity norm is large considerably. Therefore to ease the load on computation , the equivalence between the spectral radius stability criterion and the small gain theorem is obtained as:

$$\|M(s)\Delta\|_{\infty} < 1 \Longleftrightarrow \rho(M\Delta) < 1 \qquad \forall\omega \in [0,\infty) \tag{3.8}$$

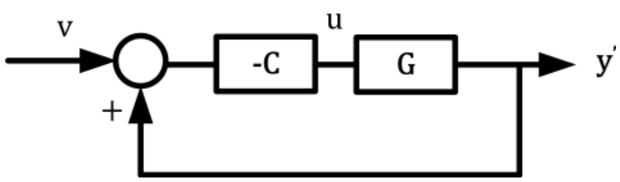

Figure 3.11: Block Diagram to find M(s)

Hence , the robust stability constraints can be rewritten respectively as:

$$\rho(C(I+GC)^{-1}) < 1 \tag{3.9}$$

$$\rho(C(I+GC)^{-1}G) < 1 \tag{3.10}$$

$$\rho(GC(I+GC)^{-1}) < 1 \tag{3.11}$$

3.4 Illustrative Example

Considering a TITO (Two-Input-Two-Output) process having time delays of a Wood-Berry Distillation Process

$$G = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{3.12}$$

The centralized controller designed from this approach:

$$C = F \cdot \begin{bmatrix} \frac{16.7s+1}{12.8(\lambda_1 s+1)} & \frac{-0.0761(16.7s+1)(14.4s+1)e^{-2s}}{(21s+1)(\lambda_2 s+1)} \\ \frac{0.0266(16.7s+1)(14.4s+1)e^{-4s}}{(10.9s+1)(\lambda_1 s+1)} & \frac{-(14.4s+1)}{19.4(\lambda_2 s+1)} \end{bmatrix} \tag{3.13}$$

where

$$F = \frac{1}{1 - \frac{0.5023(16.7s+1)(14.4s+1)}{(21s+1)(10.9s+1)}e^{-6s}} \tag{3.14}$$

As per the robust stability analysis from 3.8 for the plant with multiplicative input and multiplicative output uncertainty, from the figure 3.12

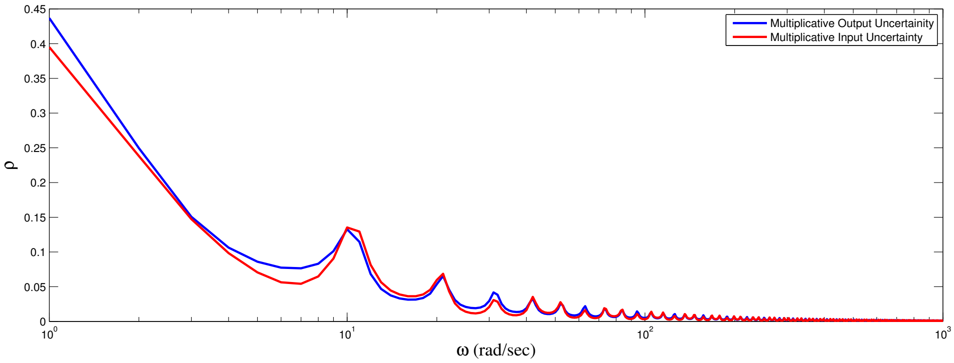

Figure 3.12: Spectral Radius Plot of Perturbed system

From the magnitude plot of the spectral radius, it was observed that as the peak value is less that unity then the system is said to have proper robust stability.

Chapter 4

Model Reduction Technique Using Gain and Phase Margin

This chapter briefly explains that the decouplers have been designed for the given system to reduce the interactions between the loops. There are certain realization problems to implement ideal and simplified decoupling [23]. Hence, we design a first order plus dead time(FOPDT) model for each of these subsystems which are decoupled based on the frequency response fitting [16].

4.1 Introduction

In general, the industrial processes are multi-variable systems in nature with a lot of interactions between the input- output process variables. Therefore, we find it difficult to tune any one loop independently. The MIMO process are controlled by decentralized controllers , or centralized controllers. The decentralized controllers are used instead of centralized controllers since it is easy to design, tune, implement and maintain. We know that the TITO system is one of the most prevailing category of the multi-variable systems. In the present work, for the TITO system a simple decoupler along with a decentralized PI/PID controller [16, 23] is has been proposed.The model order reduction technique based mostly on the frequency response fitting is used to get FOPDT model of every higher order decoupled system [14].The desired performance of the TITO system is obtained by employing a controller together with a decoupler. The controller is designed in order to fulfill the gain and phase margin. There are a wide variety of ways to design the controller [24–26].

4.2 Problem Definition

Consider a TITO (Two-Input-Two-Output) process having time delays usually identified in engineering practice as given below:

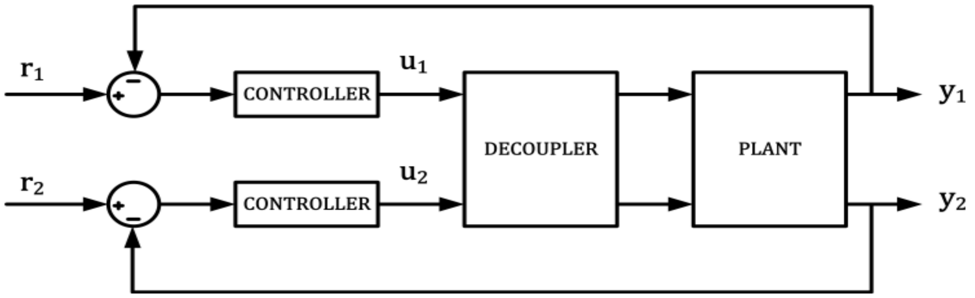

Figure 4.1: Block Diagram for Control Structure of TITO System

$$G(s) = \begin{bmatrix} g_{11}(s)e^{-\theta_{11}s} & g_{12}(s)e^{-\theta_{12}s} \\ g_{22}(s)e^{-\theta_{21}s} & g_{22}(s)e^{-\theta_{22}s} \end{bmatrix} \tag{4.1}$$

Let us assume that either diagonal or off-diagonal elements of G(s) have no right half plane (RHP) poles G(s) is decoupled with the decoupler matrix by considering the following two cases [16]:

Case 1: Suppose that the off-diagonal elements of G(s) have no RHP poles and the diagonal elements do not have any RHP zeros.

Consider the decoupler matrix as follows:

$$D(s) = \begin{bmatrix} v_1(s) & d_{12}(s)v_2(s) \\ d_{21}(s)v_1(s) & v_2(s) \end{bmatrix} \tag{4.2}$$

where $v_1(s), v_2(s), d_{12}(s)$ and $d_{21}(s)$ are as given below:

$$v_1(s) = \begin{cases} 1 & \theta_{21} \geq \theta_{22} \\ e^{(\theta_{21}-\theta_{22})s} & \theta_{21} < \theta_{22} \end{cases} \tag{4.3}$$

$$v_2(s) = \begin{cases} 1 & \theta_{12} \geq \theta_{11} \\ e^{(\theta_{12}-\theta_{11})s} & \theta_{12} < \theta_{11} \end{cases} \tag{4.4}$$

$$d_{12}(s) = -\frac{g_{12}(s)}{g_{11}(s)}e^{-(\theta_{12}-\theta_{11})s} \tag{4.5}$$

$$d_{21}(s) = -\frac{g_{21}(s)}{g_{22}(s)}e^{-(\theta_{21}-\theta_{22})s} \tag{4.6}$$

Case 2: Suppose that the diagonal elements of G(s) have no RHP poles and the off-diagonal elements do not have any RHP zeros.

Consider the decoupler matrix as follows:

$$D(s) = \begin{bmatrix} d_{11}(s)v_3(s) & v_3(s) \\ v_4(s) & d_{22}(s)v_4(s) \end{bmatrix} \tag{4.7}$$

where $v_3(s), v_4(s), d_{11}(s)$ and $d_{22}(s)$ are as given below:

$$v_3(s) = \begin{cases} 1 & \theta_{22} \geq \theta_{21} \\ e^{(\theta_{22}-\theta_{21})s} & \theta_{22} < \theta_{21} \end{cases} \tag{4.8}$$

$$v_4(s) = \begin{cases} 1 & \theta_{11} \geq \theta_{12} \\ e^{(\theta_{11}-\theta_{12})s} & \theta_{11} < \theta_{12} \end{cases} \tag{4.9}$$

$$d_{11}(s) = -\frac{g_{22}(s)}{g_{21}(s)} e^{-(\theta_{22}-\theta_{21})s} \tag{4.10}$$

$$d_{21}(s) = -\frac{g_{11}(s)}{g_{12}(s)} e^{-(\theta_{11}-\theta_{12})s} \tag{4.11}$$

Now let us consider H(s) is a diagonal matrix as given in the 4.12

$$H(s) = G(s)D(s) = diag\{h_{11}(s), h_{22}(s)\}0 \tag{4.12}$$

where $h_{ii}(s)$ are the decoupled elements which are to be controlled by the decentralized PI PID controllers $c_{ii}(s)$, where i=1,2.

4.3 Model Reduction Technique

The H(s) obtained, having the elements h_{ii}, are very complex. Therefore, there is a need to reduce it into a FOPDT model. This model describes the dead time, time constant and the process gain of the higher order processes . Here in this part FOPDT model l_{ii} of each of the elements of h_{ii} of H(s) is obtained as shown below:

$$l_{ii}(s) = \frac{K_{ii}e^{-L_{ii}s}}{T_{ii}s+1}, \qquad i = 1, 2 \tag{4.13}$$

In order to find FOPDT model of h_{ii}, there are three unknown parameters $(K_{ii}, L_{ii} and T_{ii})$ in Eq.4.13 which are to be determined.Now, on the basis of the frequency response fitting the FOPDT model is determined at two points, one at $\omega = 0$ and other at $\omega = \omega_{ci}$ [5] , where ω_{ci} is the phase crossover frequency.

$$l_{ii}(0) = h_{ii}(0) \tag{4.14}$$

$$|l_{ii}(j\omega_{ci})| = |h_{ii}(j\omega_{ci})| \tag{4.15}$$

$$\angle\{l_{ii}(j\omega_{ci})\} = \angle h_{ii}(j\omega_{ci}) \tag{4.16}$$

Using the above conditions, the FOPDT model parameters are calculated as below:

$$K_{ii} = h_{ii}(0) \tag{4.17}$$

$$T_{ii} = \sqrt{\frac{K_{ii}^2 - |h_{ii}(j\omega_{ci})|^2}{|h_{ii}(j\omega_{ci})|^2 \omega_{ci}^2}} \tag{4.18}$$

$$L_{ii} = \frac{\pi + tan^{-1}(-\omega_{ci}T_{ii})}{\omega_{ci}} \tag{4.19}$$

4.4 Controller Design

From frequency response of a system, we come to know about the phase and gain margin of the system [18]. Here in this section , the formula has been derived to calculate and design the PI or PID controllers to obtain the desired a GM and PM for a particular system.

4.4.1 PI Controller

The conditions for the gain and phase margins for any system can be represented as follows:

$$\angle\left[l_{ii}(j\omega_{pii})c_{ii}(j\omega_{pii})\right] = -\pi \tag{4.20}$$

$$A_{mii} = \frac{1}{|l_{ii}(j\omega_{pii})c_{ii}(j\omega_{pii})|} \tag{4.21}$$

$$|l_{ii}(j\omega_{gii})c_{ii}(j\omega_{gii})| = 1 \tag{4.22}$$

$$\phi_{mii} = \angle\left[l_{ii}(j\omega_{pii})c_{ii}(j\omega_{pii})\right] + \pi \tag{4.23}$$

where A_{mii} and ϕ_{mii} are the gain margin and phase margins respectively. Also, ω_{pii} and ω_{gii} is the phase crossover frequency and the gain crossover frequency. The c_{ii} is the PI controller for a FOPDT model l_{ii} as given in the Eq.4.24 below:

$$c_{ii}(s) = k_{pii}\left(1 + \frac{1}{T_{Iii}s}\right) \tag{4.24}$$

where k_{pii} refers to the proportional gain and T_{Iii} refers to the integral time.
The open loop transfer function from 4.13 and 4.24 is

$$l_{ii}(s)c_{ii}(s) = \frac{k_{pii}K_{ii}(sT_{Iii}+1)}{sT_{Iii}(sT_{ii}+1)}e^{-L_{ii}s} \tag{4.25}$$

According to the given process and their gain and phase margin specification, the crossover frequencies and the controller parameters like P and I will be determined numerically, because of the presence of the arctan function. The arctan function is approximated so as to attain an analytical solution.

$$\arctan x = \begin{cases} \frac{1}{4}x, & (|x| \leq 1) \\ \frac{1}{2}\pi - \frac{\pi}{4x}, & (|x| > 1) \end{cases} \tag{4.26}$$

Here, x may be $\omega_{pii}T_{ii}, \omega_{pii}T_{Iii}, \omega_{gii}T_{ii}$ or $\omega_{gii}T_{Iii}$

The PI controller parameter are derived as follows:

$$k_{pii} = \frac{\omega_{pii}T_{ii}}{A_{mii}K_{ii}}, \qquad T_{Iii} = \left(2\omega_{pii} - \frac{4\omega_{pii}^2 L_{ii}}{\pi} + \frac{1}{T_{ii}}\right)^{-1} \tag{4.27}$$

here,

$$\omega_{pii} = \frac{A_{mii}\phi_{mii} + \frac{1}{2}\pi(A_{mii} - 1)}{(A_{mii}^2 - 1)L_{ii}} \tag{4.28}$$

4.4.2 PID Controller

The PID controller in a series form with a first order filter is as follows:

$$c_{ii}(s) = \frac{k'_{pii}(sT'_{Iii} + 1)(sT'_{Dii} + 1)}{sT'_{Iii}(sT'_{Fii} + 1)} \tag{4.29}$$

where $k'_{pii}, T'_{Dii}, T'_{Iii}$ and T'_{Fii} are the proportional gain, derivative time, integral time and filter time constant respectively.

In the parallel form of PID controller, T'_{Dii} is commonly chosen as a ratio of the integral time T'_{Iii} ($T_{Dii} = \frac{1}{4}T_{Iii}$). Similarly,for the series PID controller as given in Eq.4.29, the condition will be [27–29]:

$$T'_{Dii} = T'_{Iii} \tag{4.30}$$

Comparing 4.13, 4.29 and 4.30, the open loop transfer function as follows.

$$l_{ii}(s)c_{ii}(s) = \frac{K_{ii}k'_{Pii}(sT'_{Iii} + 1)(sT'_{Dii} + 1)}{sT'_{Iii}(sT_{ii} + 1)(sT'_{Fii} + 1)} e^{-L_{ii}s} \tag{4.31}$$

Let $T'_{Fii} = T'_{Iii}$, then the 4.31 is simplified as

$$l_{ii}(s)c_{ii}(s) = \frac{K_{ii}k'_{Pii}(sT'_{Iii} + 1)}{sT'_{Iii}(sT_{ii} + 1)} e^{-L_{ii}s} \tag{4.32}$$

Therefore, PID controller parameters is calculated,where ω_{pii} is from 4.28:

$$k'_{Pii} = \frac{\omega_{pii}T_{ii}}{A_{mii}K_{ii}}, \qquad T'_{Iii} = \left(2\omega_{pi} - \frac{4\omega_{pi}^2 L_{ii}}{\pi} + \frac{1}{T_{ii}}\right)^{-1} \tag{4.33}$$

$$T'_{Dii} = T'_{Iii} \qquad T'_{Fii} = T'_{Iii} \tag{4.34}$$

Chapter 5

Decoupling Controller Design for Coupled Tank System Model

5.1 Introduction

The level control problem is an important characteristic which is featured as a benchmark problem in a coupled tank system categorized in nonlinear and unstable control system. In various most of the process industries such as paper making industries, petro-chemical industries, water treatment industries,etc. the fluid level and flow control is essential. Figure 5.1 illustrates some of the industrial applications of coupled tank system.

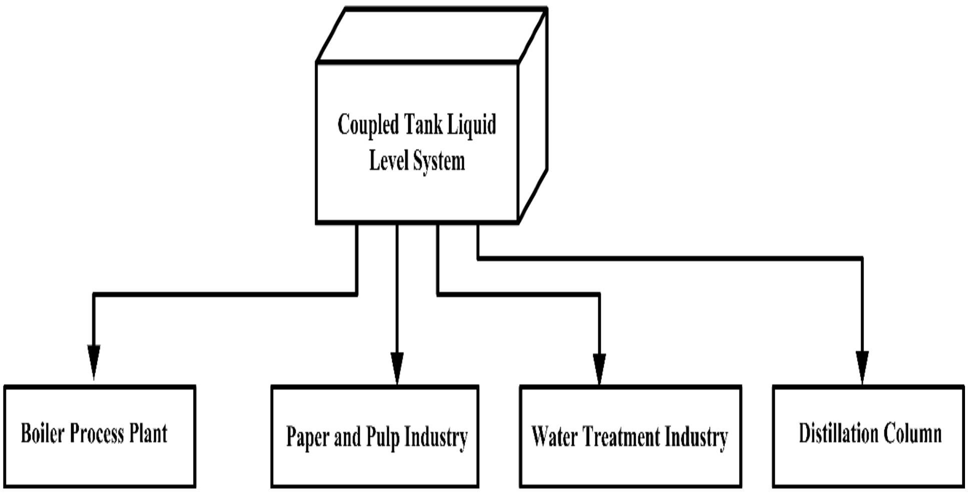

Figure 5.1: Coupled Tank System Applications

In process industries, managing the liquid levels in s tanks and the liquid flowing within them is a standard problem. Initially, the liquid has to be pumped in to a tank and stored in it, thereby leading the liquid to flow into other tanks in the process industries. The liquids will be processed many times by mixing treatment or chemicals in the tank, but it is important that the level of the liquid in the tank is controlled always and the fluid flow must be regulated. Quite often the tanks are coupled together such that there is interaction between the liquid levels and this must also be controlled.

The schematic representation of a liquid level system is as shown below in figure 5.2 :

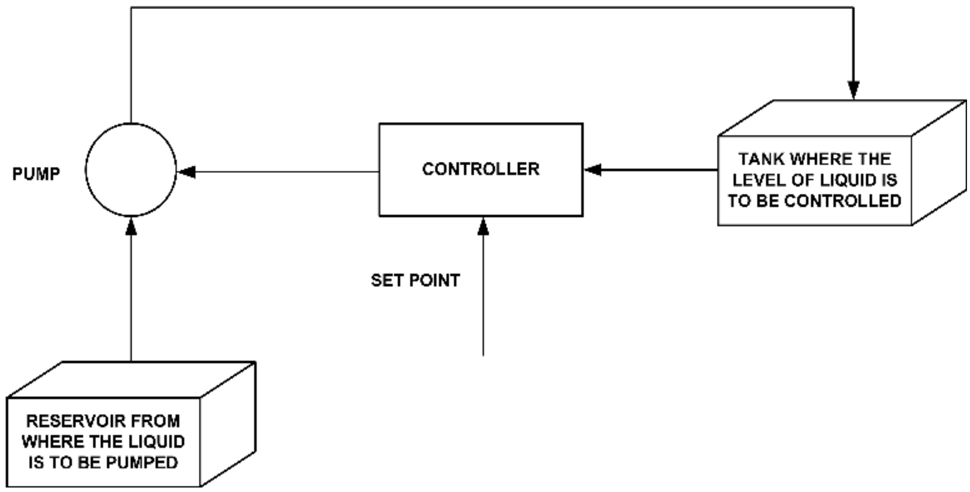

Figure 5.2: Block Diagram of Liquid Level System

The coupled tank system is an example of a multi-variable system. The input to the tank is the control voltage , whereas the output is the level of water in the tank. The main aim of the coupled tank system is to sustain the desired liquid level of the tank when the water flows into and out of the tank . This system has a to face few challenges because of its certain characteristics like the large time-delay, non- linearity and non-minimum phase zeros.

5.2 Description of Coupled Tank System

The coupled tank system comprises of four translucent tanks.These tanks are equipped with a pipe in the outlet so that the water which overflows is transmitted to the reservoir. In this system, tank 5 that is at the bottom acts as reservoir that used for storing water. In this setup, below each of the four tanks a level sensor is provide. The sensor is actually a transducers which with the assistance of signal conditioning circuit convert the water level, that is the output, into DC voltage (0-5 volts) .This sensor measures the level of water in the tanks. The water level is monitored by a scale that is attached to all the individual tanks. The reservoirs contains water which has to be pumped up to the tanks. The coupled tank system works on the basis of two basic modes:

1. Local Mode
2. Remote mode

In the local mode, water is driven to the respective tanks with the help of two separate potentiometer that are applied to the two tanks, and these potentiometers also control them. The interactions present in the system is represented by the coupling probe in between the tanks. The water level varies as the water starts to flow from higher to lower level of the tank. The present work mainly focuses on the local mode of operation of the coupled tank and it is represented as TITO (Two - Input Two - output) system.

The schematic representation of the coupled tank system is as shown in figure 5.3 below:

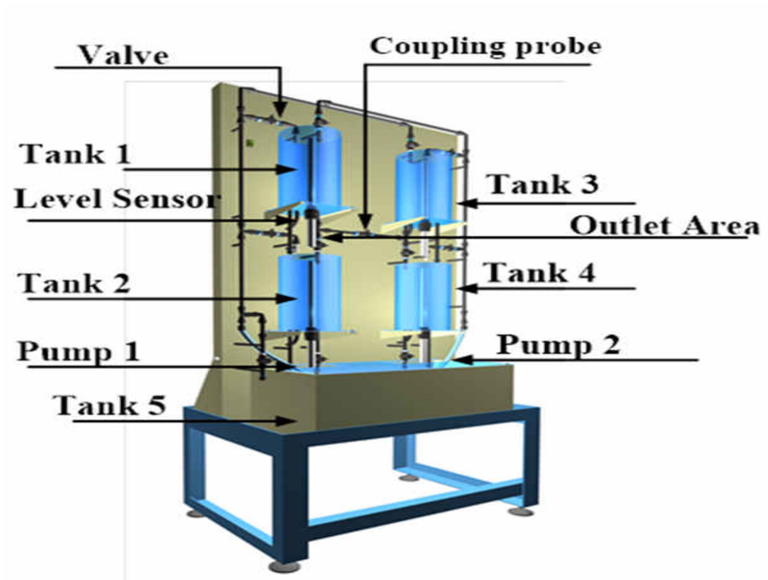

Figure 5.3: Coupled Tank System Mechanical Unit
[30]

5.3 Experimental Setup of Coupled Tank System

The coupled tank system is outfitted with power amplifier (PSUPA) , cable connector box and a power supply unit. The experimental setup is as shown in figure. The main control unit of this setup is MATLAB/SIMULINK environment and PC with Advantech. The water pressure level signals are amplified and are the passed on as analogue signals to the PCI1711 DAQ card by the PSUPA units. The pc sends control signals to the pump through the PSUPA unit and the DAQ (PSUPA). The PSUPA unit that convey the control signals, which are between 0-5 volt, change them to 24V PWM signs to drive the pumps.

5.4 Problem Definition

The transfer function model can be obtained by the system identification for the coupled tank system. Here the mode of operation is the local mode.The transfer function is given as:

$$G(s) = \begin{bmatrix} \dfrac{2.197e^{-5.5s}}{615s+1} & \dfrac{2.3e^{-9.34s}}{614s+1} \\ \dfrac{2.197e^{-30s}}{601.84s+1} & \dfrac{2.8e^{-30s}}{602s+1} \end{bmatrix} \tag{5.1}$$

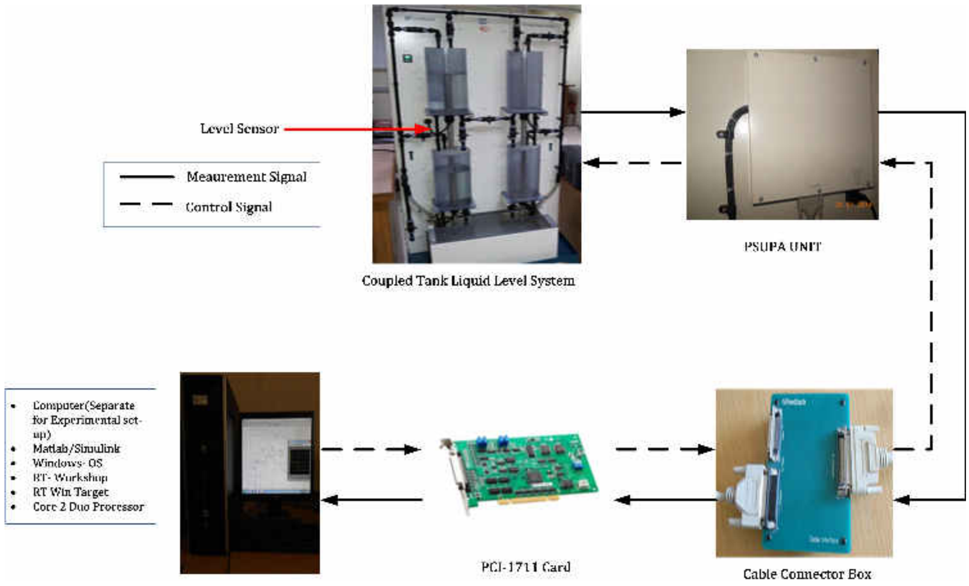

Figure 5.4: Experimental Setup Coupled Tank System
[30]

The designing of the decoupling controller for achieving the desired liquid level in tank 1 and 3, when the water flows into and out of the tank, is the control problem of this system. In the presence of coupling probe, the fluid level of tank 1 and 3 must not depend on the change in the required liquid level of any of the tank.

5.4.1 Decoupler Design By Gain Margin and Phase Margin Specification

- The decoupler design for the above system 5.1 , considering the case where off-diagonal elements of G(s) have no RHP poles and the diagonal elements do not have any RHP zeros. Then, $\theta_{21} = \theta_{22} = 30$ and $\theta_{12} > \theta_{11}$ then according to equations 4.2,4.4, 4.5, 4.6 and 4.6, the decoupler obtained will be:

$$D(s) = \begin{bmatrix} 1 & -\frac{G_{12}(s)}{G_{11}(s)} \\ -\frac{G_{21}(s)}{G_{22}(s)} & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\dfrac{1.047e^{3.84s}(615s+1)}{(614s+1)} \\ -\dfrac{0.7846(602s+1)}{(601.84s+1)} & 1 \end{bmatrix} \tag{5.2}$$

- Then the H(s), diagonal TFM as given in 4.12

$$h_{11}(s) = \frac{2.197e^{-5.5s}}{(615s+1)} - \frac{1.80458(602s+1)e^{-9.34s}}{(614s+1)(601.84s+1)} \tag{5.3}$$

$$h_{22}(s) = \frac{2.8e^{-30s}}{(602s+1)} - \frac{2.3(615s+1)e^{-33.84s}}{(614s+1)(601.84s+1)} \tag{5.4}$$

Then solving 4.18,4.19 and 4.19 we get :

- Then solving 4.18,4.19 and 4.19 we get :
  - For $h_{11}(s)$with phase crossover frequency $0.411 rad/sec$:

$$K_{11} = 0.3924$$
$$T_{11} = 620.765$$
$$L_{11} = 3.831$$

  - For $h_{22}(s)$ with phase crossover frequency $0.0818 rad/sec$:

$$K_{11} = 0.5$$
$$T_{11} = 632.74$$
$$L_{11} = 19.45$$

- The resultant FOPDT model of the diagonal subsystems are as follows :

$$l_{11}(s) = \frac{0.3924 e^{-3.831s}}{620.765s + 1} \tag{5.5}$$
$$l_{22}(s) = \frac{0.5 e^{-19.45s}}{632.74s + 1} \tag{5.6}$$

Assume that $A_{m11} = 5, A_{m22} = 3.5, \phi_{mii} = 60\,\text{deg}$. The controller designed will be

$$\begin{bmatrix} 39.644 + \frac{0.0639}{s} & 0 \\ 0 & 12.546 + \frac{0.0198}{s} \end{bmatrix} \tag{5.7}$$

The simulation result is as shown below : The method described in chapter-4, IMC method

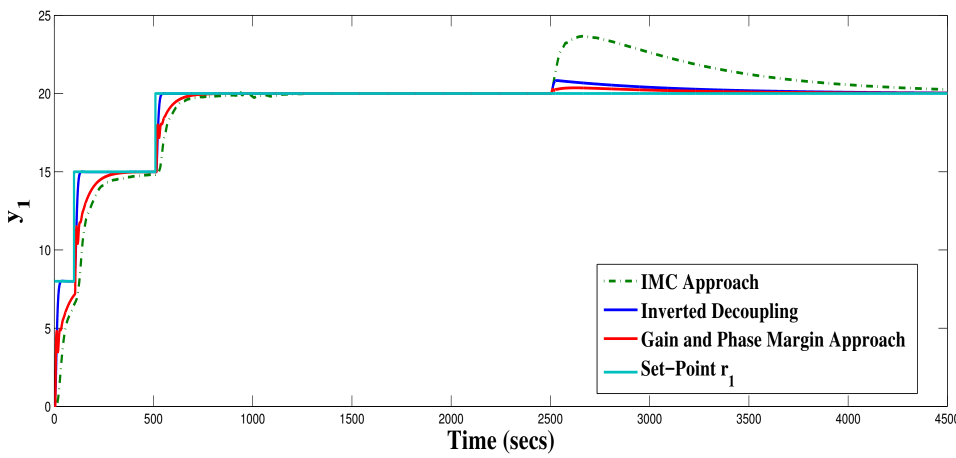

Figure 5.5: Output response of y_1

[10] and inverted decoupling technique was applied to the coupled tank system model.

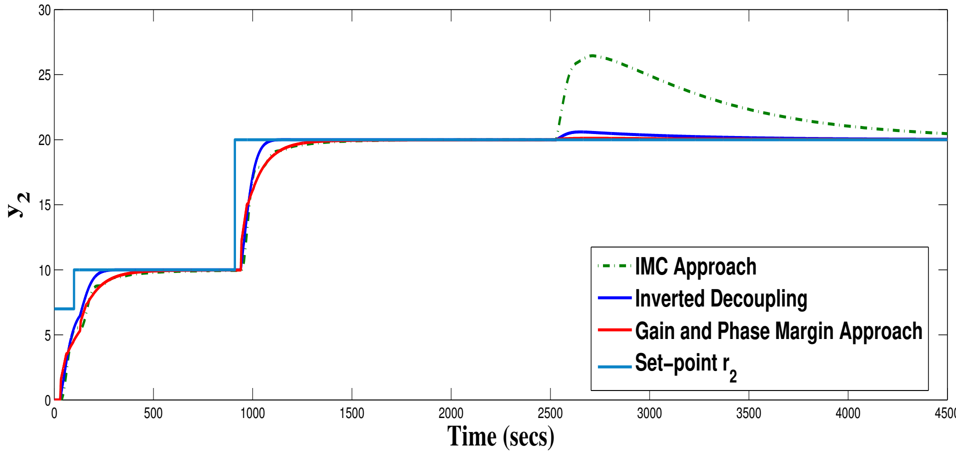

Figure 5.6: Output response of y_2

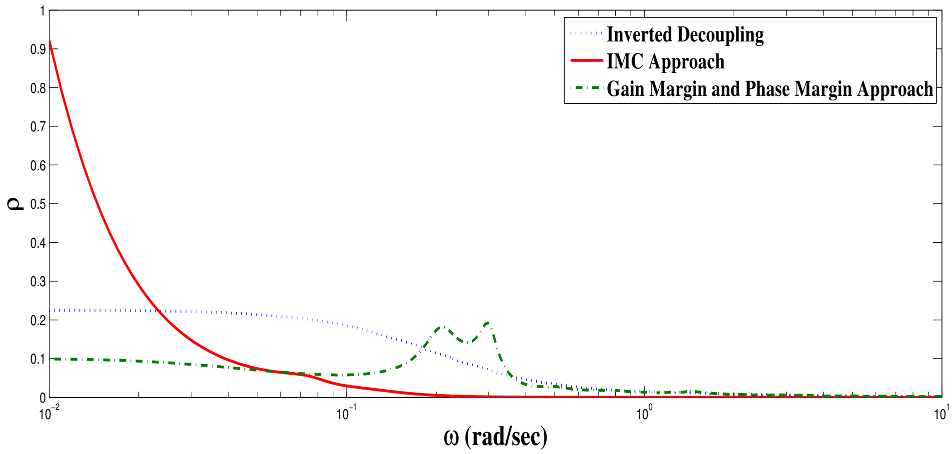

Figure 5.7: Perturbed plant model with Multiplicative uncertainty

- At first, a step input having amplitude of 8 units was applied to the first tank at $t = 0$ seconds upto $t = 100$ seconds , after that a step change of amplitude 7 was applied upto $t = 510$ seconds and later a step change of amplitude 5 was given upto $t = 4500$seconds .
- At first, a step input having amplitude of 7 units was applied to the first tank at $t = 0$ seconds upto $t = 100$ seconds , after that a step change of amplitude 3 was applied upto $t = 910$ seconds and later a step change of amplitude 100 was given upto $t = 4500$ seconds . These step changes are referred as set-points for the system.
- These set - points represent the water level in the tank like amplitude of 7 means 7cm water level.
- It was observed that inverted decoupling technique has faster set-point tracking than the IMC approach and gain margin and phase margin approach.

- The objective of decoupling is achieved by all the approaches.
- The gain margin and phase margin approach provides a proper disturbance rejection than the inverted decoupling and IMC approach.
- The tuning parameter λ is a reason for slower response of IMC approach.
- Figure 5.7 shows that the magnitude of spectral radius plot is less than 1 for multiplicative output uncertainty, hence the three decoupling methods provide good robust stability by the spectral radius criteria.

Chapter 6

Conclusion and Future Scope

6.1 Conclusion

From the model reduction technique using gain and phase Margin as described in chapter 4, it is observed that from this method , one can reduce the diagonal transfer matrix H(s) into FOPDT model thus reducing the complexity in calculation of finding the PI/PID controller parameter. It was observed that this method gives better performance against disturbance rejection, but the set point tracking is not adequate. On the other hand, the inverted decoupling technique yields better set-point tracking, however the disturbance rejection in not satisfactory.

6.2 Future scope

- It is observed that, in the model reduction technique the set-point tracking was not suitable though the disturbance rejection. Further modifications can be made by designing of H_2 control so as to obtain satisfactory set-point tracking as well as disturbance rejection.
- An LPV based model can be further design to obtain adequate model for coupled tank system.

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